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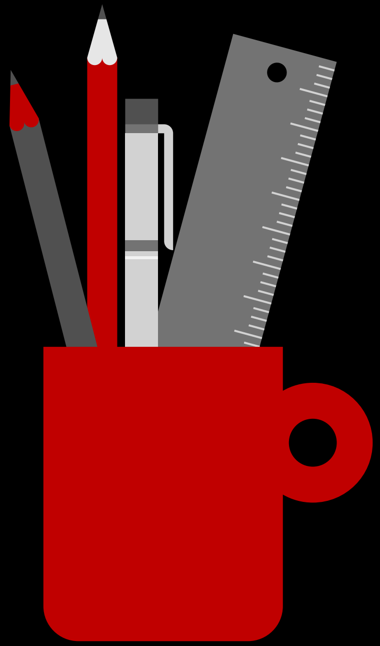

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Review Ex. 4

3) $x + \frac{1}{x} = 3$ (i) $x^2 + \frac{1}{x^2} = ?$ (ii) $x^4 + \frac{1}{x^4} = ?$

Sol:-

$x + \frac{1}{x} = 3$

$\left(x + \frac{1}{x}\right)^2 = (3)^2$

$x^2 + \frac{1}{x^2} + 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = 9$

$x^2 + \frac{1}{x^2} = 9 - 2$

$\boxed{x^2 + \frac{1}{x^2} = 7}$

$x^2 + \frac{1}{x^2} = 7$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$

$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right) = 49$

$x^4 + \frac{1}{x^4} = 49 - 2$

$\boxed{x^4 + \frac{1}{x^4} = 47}$

4) $x - \frac{1}{x} = 2$ (i) $x^2 + \frac{1}{x^2} = ?$ ii) $x^4 + \frac{1}{x^4} = ?$

Sol:-

$$x - \frac{1}{x} = 2$$

$$\left(x - \frac{1}{x}\right)^2 = (2)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 - 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = 4$$

$$x^2 + \frac{1}{x^2} - 2 = 4$$

$$x^2 + \frac{1}{x^2} = 4 + 2$$

$$\boxed{x^2 + \frac{1}{x^2} = 6}$$

$$x^2 + \frac{1}{x^2} = 6$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (6)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right) = 36$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$\boxed{x^4 + \frac{1}{x^4} = 34}$$

5) $x^3 + y^3 = ?$ $xy = ?$ $x + y = 5, x - y = 3$

Sol:

$$4xy = (x+y)^2 - (x-y)^2$$

$$4xy = (5)^2 - (3)^2$$

$$4xy = 25 - 9$$

$$\frac{\cancel{4}xy}{\cancel{4}} = \frac{\cancel{16}^{\,4}}{\cancel{4}}$$

$$\boxed{xy = 4}$$

$$x + y = 5$$

$$(x+y)^3 = (5)^3$$

$$x^3 + y^3 + 3xy(x+y) = 125$$

$$x^3 + y^3 + 3(4)(5) = 125$$

$$x^3 + y^3 + 60 = 125$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65 \text{ Ans.}$$

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6) $\rho = 2+\sqrt{3}$ $\quad \rho+\frac{1}{\rho}=?$ $\quad \rho-\frac{1}{\rho}=?$ $\quad \rho^2+\frac{1}{\rho^2}=?$ $\quad \rho^2-\frac{1}{\rho^2}=?$

sol:

$$\rho = 2+\sqrt{3}$$

$$\frac{1}{\rho} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{\rho} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$\frac{1}{\rho} = \frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{\rho} = 2-\sqrt{3}$$

$$\rho+\frac{1}{\rho} = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$$

$$\boxed{\rho+\frac{1}{\rho} = 4}$$

$$\rho-\frac{1}{\rho} = (2+\sqrt{3})-(2-\sqrt{3})$$

$$\rho-\frac{1}{\rho} = \cancel{2}+\sqrt{3}\cancel{-2}+\sqrt{3}$$

$$\boxed{\rho-\frac{1}{\rho} = 2\sqrt{3}}$$

$$\rho+\frac{1}{\rho} = 4$$

$$\left(\rho+\frac{1}{\rho}\right)^2 = (4)^2$$

$$\rho^2+\frac{1}{\rho^2}+2(\cancel{\rho})\left(\frac{1}{\cancel{\rho}}\right) = 16$$

$$\rho^2+\frac{1}{\rho^2} = 16-2$$

$$\boxed{\rho^2+\frac{1}{\rho^2} = 14}$$

$$\rho^2-\frac{1}{\rho^2} = \left(\rho+\frac{1}{\rho}\right)\left(\rho-\frac{1}{\rho}\right)$$

$$= (4)(2\sqrt{3})$$

$$\boxed{\rho^2-\frac{1}{\rho^2} = 8\sqrt{3}}$$

7) $q = \sqrt{5} + 2$ $\quad q + \frac{1}{q} = ?$ $\quad q - \frac{1}{q} = ?$ $\quad q^2 + \frac{1}{q^2} = ?$ $\quad q^2 - \frac{1}{q^2} = ?$

Sol:

$$q = \sqrt{5} + 2$$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{(\sqrt{5})^2 - (2)^2}$$

$$\frac{1}{q} = \frac{\sqrt{5}-2}{5-4}$$

$$\frac{1}{q} = \sqrt{5} - 2$$

$$q + \frac{1}{q} = \sqrt{5} \cancel{+2} + \sqrt{5} \cancel{-2}$$

$$\boxed{q + \frac{1}{q} = 2\sqrt{5}}$$

$$q - \frac{1}{q} = (\sqrt{5}+2) - (\sqrt{5}-2)$$

$$= \cancel{\sqrt{5}} + 2 \cancel{-\sqrt{5}} + 2$$

$$\boxed{q - \frac{1}{q} = 4}$$

$$q - \frac{1}{q} = 4$$

$$\left(q - \frac{1}{q}\right)^2 = (4)^2$$

$$q^2 + \frac{1}{q^2} - 2(\cancel{q})\left(\frac{1}{\cancel{q}}\right) = 16$$

$$q^2 + \frac{1}{q^2} = 16 + 2$$

$$\boxed{q^2 + \frac{1}{q^2} = 18}$$

$$q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right)\left(q - \frac{1}{q}\right)$$

$$= (2\sqrt{5})(4)$$

$$\boxed{q^2 - \frac{1}{q^2} = 8\sqrt{5}}$$

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8 (i)

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{\cancel{a^2}+2-\cancel{a^2}+2}$$

$$=\frac{a^2\cancel{+2}+a^2\cancel{-2}+2\sqrt{a^4-4}}{4}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}\,2}$$

$$=\boxed{\frac{a^2+\sqrt{a^4-4}}{2}}$$

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8) ii) $\dfrac{1}{a-\sqrt{a^2-x^2}} - \dfrac{1}{a+\sqrt{a^2-x^2}}$

$$= \frac{\left(a+\sqrt{a^2-x^2}\right) - \left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)\left(a+\sqrt{a^2-x^2}\right)}$$

$$= \frac{\cancel{a}+\sqrt{a^2-x^2}\cancel{-a}+\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^{\cancel{2}}}$$

$$= \frac{2\sqrt{a^2-x^2}}{\cancel{a^2}\cancel{-a^2}+x^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{x^2} \quad \text{Ans.}$$

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